Harvesting from a Private Lake (Soc. Opt.):

$B(Q_t)$ is current payoff (area under the demand curve) for catch Q_t.
$C(Q_t, S_t)$ is current harvest cost for Q_t given stock S_t in the lake (larger S_t means lower cost to catch Q_t).
Organize sequence of harvests $Q_0, Q_1, Q_2, ...,$ to max *PV*.

$$W = B(Q_0) - C(Q_0, S_0) + \left(\frac{1}{1+r}\right)\left\{B(Q_1) - C(Q_1, S_1)\right\} + \left(\frac{1}{1+r}\right)^2 \left\{B(Q_2) - C(Q_2, S_2)\right\} + ...$$

Where, $Q_0 = S_1 - S_0$
$Q_1 = S_2 - S_1$

AND stock grows naturally as $S_{t+1} = S_t + \varphi(S_t) - Q_t$

(if current harvest Q_t equals natural increment $\varphi(S_t)$, then $S_t = S_{t+1}$ and stock size is unchanging.)

$$W = B(S_0 + \varphi(S_0) - S_1) - C(S_0 + \varphi(S_0) - S_1, S_0) + \left(\frac{1}{1+r}\right)\left\{B(S_1 + \varphi(S_1) - S_2) - C(S_1 + \varphi(S_1) - S_2, S_1)\right\}$$
$$+ \left(\frac{1}{1+r}\right)^2 \left\{B(S_2 + \varphi(S_2) - S_3) - C(S_2 + \varphi(S_2) - S_3, S_2)\right\} + ...$$

$\frac{\partial W}{\partial S_1} = 0$ implies,

$$-[B_{Q_0} - C_{Q_0}] + \left(\frac{1}{1+r}\right)[B_{Q_1} - C_{Q_1}] + \left(\frac{1}{1+r}\right)Q_{S_1}[B_{Q_1} - C_{Q_1}] - \left(\frac{1}{1+r}\right)C_{S_1} = 0$$

$$p(Q_1) - MC(Q_1) - [p(Q_0) - MC(Q_0)] + [p(Q_1) - MC(Q_1)]\varphi_{S_1} - C_{S_1} = r[p(Q_0) - MC(Q_0)] \quad (*)$$

This equation works for any consecutive periods *t*, *t*+1 and can drive the system form historically given stock S_0 to a “final” stock S_∞.

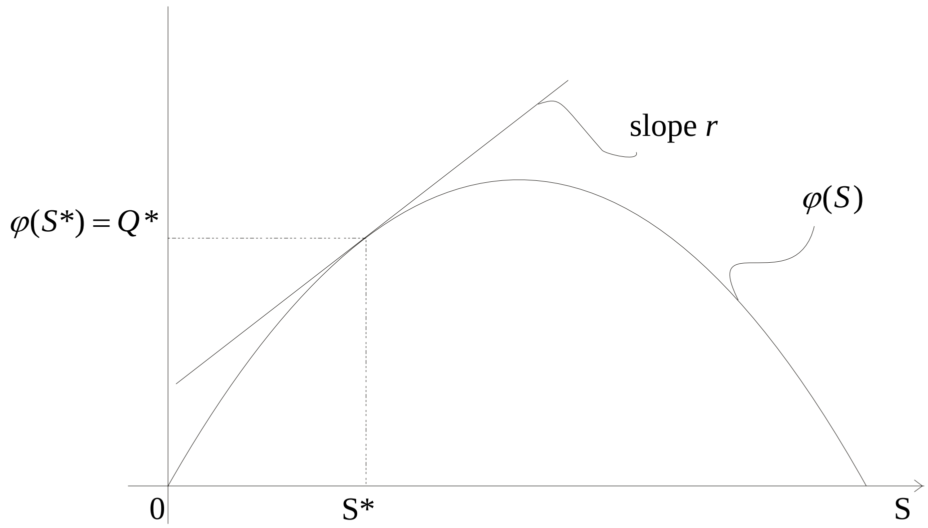
Case 1: at a steady state, $Q_t = Q_{t+1}$, and $S_t = S_{t+1}$ then (*) reduces to,

$$[p(Q^*) - MC(Q^*)]\varphi_{S^*} - C_{S_1^*} = r[p(Q^*) - MC(Q^*)]$$

Case 2: at a steady state and harvest costs are assumed simple (i.e. $C_S = 0$).

Then $\varphi_{S^*} = r$

Biological interest rate equals market rate.


slope r
$\varphi(S)$
$\varphi(S^*) = Q^*$
0
S*
S